Multiple Extended Object Tracking Using PMHT with Extension-Dependent Measurement Numbers

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***Abstract*—For multiple extended object tracking (MEOT), data association and object extension estimation are key problems, and the number of measurements generated by each object plays a key role in both problems. For data association, probabilistic multiple hypothesis tracking (PMHT) naturally assumes multiple measurements can be assigned to a single object and has a linear computation complexity, and thus it fits MEOT well. In existing PMHT approaches to MEOT, the measurement number is usually used for data association only, not for direct extension estimation. Since the measurement number contains the extension information, e.g., an object with a bigger extension tends to generate more measurements given the sensor resolution, utilizing the measurement number for extension estimation is expected to improve the tracking performance. This paper proposes a PMHT approach combined with a random-matrix model using extension-dependent measurement numbers. The proposed approach derives a new auxiliary function of which the likelihood function part reflects the extension information contained in the measurement number. Then, using Expectation-Maximization, analytical forms of iteration formulae for kinematic states and extensions of multiple objects can be obtained approximately. Since more information is considered, the tracking performance of MEOT is improved, especially in extension estimation. Simulation results demonstrate the effectiveness of the proposed approach.**

***Index Terms*—Multiple Extended Object Tracking, Probabilistic Multiple Hypothesis Tracking, Extension-Dependent Measurement Number, Random Matrix, Expectation-Maximization**

I. Introduction

On account of increased radar resolution, multiple measurements can be obtained for a single object per scan [1]. The number of measurements generated by an object depends on the object extension, the sensor-to-object geometry, and the radar resolution [2]. Based on multiple measurements, not only the kinematic state but also the extension of an object can be estimated. Such an object is usually called an extended object (EO). For extension estimation, it is not trivial because the correspondence between a measurement and a measurement source generating it cannot be distinguished easily [3]. Moreover, in practice, there are usually multiple objects in the surveillance area. The simultaneous estimation of the kinematic state and the extension of each object is referred to as multiple extended object tracking (MEOT). For MEOT, the assignment of measurements to objects, i.e., data association, needs to be conducted due to the measurement origin uncertainty. Data association in MEOT is also complicated since each EO can produce multiple measurements, of which the number depends on the object extension. In conclusion, extension estimation and data association are key problems in MEOT and make MEOT challenging.

For extension estimation, there are abundant approaches based on diverse models, such as the random-matrix model (RMM) [1], the random-hypersurface model [4], [5], the Gaussian process model [6], etc. Therein, a Bayesian approach using the RMM was first proposed in [1] to jointly estimate the kinematic state and the elliptical extension of an EO. Because of the concise recursion form, the RMM appears promising and several further developments have been proposed in [3], [7], [8], [9], [10], [11], [12], etc. In the above RMM-based approaches, estimations of the kinematic state and the extension are usually based on the statistics of measurements [2]. However, as described in [2], the measurement number also contains the object extension information since the number is related to the extension, the sensor-to-object geometry, and sensor resolution. Based on the relation, a novel RMM-based approach is proposed in [2] to improve the tracking performance using not only the statistics but also the number of measurements. To our knowledge, the approach in [2] is the first one that directly utilizes the measurement number to estimate the object extension.

For data association, there are also lots of algorithms, such as joint probabilistic data association (JPDA) [13], multiple hypothesis tracking (MHT) [14], [15], probabilistic multiple hypothesis tracking (PMHT) [16], etc. Therein, PMHT has a linear computation complexity [18] and adopts a soft decision for data association. In particular, it assumes assignments of measurements to objects as stochastically independent variables [16]. This assumption implies that multiple measurements can be assigned to the same object [18], and thus it fits MEOT naturally. Under this assumption and given the object number [16], the kinematic state and the extension of each object can be estimated iteratively using Expectation-Maximization (EM). For each iteration, all variables are updated to maximize the auxiliary function (Q-function). In

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theory, the auxiliary function should contain all available information including the object dynamics and the measurement process. However, for existing PMHT approaches to MEOT, such as those combined with RMMs [17], [18], only the information of object dynamics and measurement values is contained in the auxiliary function, whereas extension-dependent measurement numbers are not contained explicitly. That is, the number of measurements generated by each object is not directly used for estimation in existing PMHT approaches to MEOT.

This paper proposes a PMHT approach combined with a RMM using extension-dependent measurement numbers for MEOT. The measurement number of each object is important for both data association and extension estimation in MEOT. For data association, its main difficulty arises from the unknown measurement number of each object, where the number depends on the object extension. For extension estimation, the measurement number itself contains the extension information. Therefore, the tracking performance of MEOT is expected to be improved by utilizing extension-dependent measurement numbers. In our work, to utilize extension-dependent measurement numbers, the RMM proposed in [2] is incorporated into the PMHT framework. First, a new auxiliary function is derived where extension-dependent measurement numbers are reflected in the likelihood function part. Then, using an approximation, the likelihood function in the auxiliary function has the same form as that in [2], and thus the analytical forms of the iteration formulae for the kinematic state and the extension of each object can be obtained easily. Based on the above, a PMHT approach to MEOT with extension-dependent measurement numbers is proposed. The effectiveness of the proposed PMHT approach is demonstrated by simulation results, compared with existing PMHT approaches which are combined with other RMMs.

This paper is organized as follows: Section II reviews RMMs and the PMHT logic. Section III derives a new auxiliary function considering extension-dependent measurement numbers and proposes a new PMHT approach to MEOT based on it. Simulation results are presented in Section IV, and Section V concludes this paper.

II. Reviews

A. Random-matrix models

In the RMM, the kinematic state of an object at time k is denoted by a random vector x_k, e.g., $x_k = [p_k^{\mathrm{T}}, \dot{p}_k^{\mathrm{T}}]^{\mathrm{T}}$ with the positional state component p_k. The elliptical extension is modeled by a symmetric positive definite (SPD) random matrix $X_k \in \mathbb{R}^{d\times d}$ with the spatial dimension d. The conditional probability density function (PDF) is assumed as [1]

$$\begin{aligned} p[x_k, X_k|Z^k] &= p[x_k|X_k, Z^k]p[X_k|Z^k] \\ &= \mathcal{N}(x_k; \hat{x}_{k|k}, P_{k|k} \otimes X_k)\mathcal{IW}(X_k; \hat{v}_{k|k}, \hat{X}_{k|k}) \end{aligned} \tag{1}$$

where $Z^k = Z_{1:k}$, and $Z_k = \{z_k^1, ..., z_k^{n_k}\}$ denotes the set of n_k measurements at time k; $\mathcal{N}(m, \Sigma)$ denotes the normal distribution with mean m and covariance matrix Σ; "$\otimes$" stands for the Kronecker product; $\mathcal{IW}(Y; a, C)$ denotes the inverse Wishart distribution defined as [19]

$$\mathcal{IW}(Y; a, C) = c^{-1}|C|^{\frac{a-d-1}{2}}|Y|^{-\frac{a}{2}}\mathrm{etr}(-CY^{-1}/2) \tag{2}$$

where $C \in \mathbb{R}^{d\times d}$ and $Y \in \mathbb{R}^{d\times d}$ both denote SPD matrices; $a > 2d$ is the degree of freedom; etr stands for $\exp(\mathrm{trace}(\cdot))$, and c is a normalization factor.

The dynamic model for the kinematic state is given as [1]

$$x_k = \Phi_k x_{k-1} + w_k, \quad w_k \sim \mathcal{N}(\mathbf{0}, D_k \otimes X_k) \tag{3}$$

where $\Phi_k = F_k \otimes I_d$ with F_k denoting the dynamic matrix in the one-dimensional physical space, and $I_d \in \mathbb{R}^{d\times d}$ is an identity matrix; $\mathbf{0}$ is a zero vector of an appropriate dimension; D_k can be viewed as a coefficient of the covariance matrix of the independent process noise w_k, e.g., $D_k = \sigma_a^2[dt^2/2, dt]^{\mathrm{T}}[dt^2/2, dt]$ with the sampling interval dt and the sigma of the acceleration noise σ_a.

Assuming the extension is nearly time-invariant, the dynamic model for the extension is modeled as [1]

$$p[X_k|X_{k-1}] = \mathcal{W}(X_k; \delta_{k|k-1}, X_{k-1}/\delta_{k|k-1}) \tag{4}$$

where $\delta_{k|k-1}$ denotes the degree of freedom; $\mathcal{W}(Y; a, C)$ with $a > d-1$ is the Wishart distribution for the SPD matrix $Y \in \mathbb{R}^{d\times d}$, and it's defined as [19]

$$\mathcal{W}(Y; a, C) = c^{-1}|C|^{-\frac{a}{2}}|Y|^{\frac{(a-d-1)}{2}}\mathrm{etr}(-C^{-1}Y/2). \tag{5}$$

To describe the complex extension variation, the dynamic model for the extension X_k is extended as [3]

$$p[X_k|X_{k-1}] = \mathcal{W}(X_k^t; \delta_k, A_k X_{k-1} A_k^{\mathrm{T}}) \tag{6}$$

where $A_k \in \mathbb{R}^{d\times d}$ describes the specific evolution mode of the extension, e.g., a rotation.

The measurement model is assumed as [1]

$$z_k^r = \tilde{H}_k x_k + v_k^r, \quad r = 1, ..., n_k \tag{7}$$

where z_k^r denotes the r-th measurement at time k; $\tilde{H}_k = H_k \otimes I_d$ with H_k denoting a measurement matrix in the one-dimensional space, e.g., $H_k = [1, 0]$ for the position measurement; v_k^r denotes an independent observation noise, and it is postulated as $v_k^r \sim \mathcal{N}(\mathbf{0}, X_k)$ in [1]. To describe the observation distortion in practice, it's further extended as [3]

$$v_k^r \sim \mathcal{N}(\mathbf{0}, B_k X_k B_k^{\mathrm{T}}) \tag{8}$$

where $B_k \in \mathbb{R}^{d\times d}$ describes the distortion of the observed extension.

Recently, to model the dependency between the object extension X_k and the measurement number n_k, a Gamma-alike distribution has been proposed as [2]

$$\begin{aligned} p(n_k; \mu_k, X_k) &\triangleq \frac{(n_k)^{\mu_k - 1}}{\Gamma(\mu_k)\theta_k^{\mu_k}}\exp(-n_k a_k \beta_k), \\ \beta_k &\triangleq \mu_k \mathrm{tr}(\hat{X}_{k|k-1}X_k^{-1})/g_k, \\ \theta_k &\triangleq g_k|X_k|^{1/2}/\mu_k, \\ a_k &\triangleq 1/\mathrm{E}[\theta_k\beta_k|Z^{k-1}] \end{aligned} \tag{9}$$

where μ_k denotes the shape parameter; g_k relates the object volume with the average measurement number as in [2].

Based on the measurement models (7), (8), and (9), the overall likelihood function at time k can be given as [2]

$$\begin{aligned} p[Z_k|x_k, X_k] &= p[Z_k|n_k, x_k, X_k] \times p[n_k|x_k, X_k] \\ &\propto N(\bar{z}_k; \tilde{H}_k x_k, B_k X_k B_k^{\mathrm{T}}/n_k) \\ &\times W(\bar{Z}_k; n_k - 1, B_k X_k B_k^{\mathrm{T}}) \times p(n_k; \mu_k, X_k) \end{aligned} \tag{10}$$

with

$$\bar{z}_k = \frac{1}{n_k}\sum_{r=1}^{n_k} z_k^r,\ \bar{Z}_k = \sum_{r=1}^{n_k}(z_k^r - \bar{z}_k)(z_k^r - \bar{z}_k)^{\mathrm{T}}. \tag{11}$$

In conclusion, given parameters of $p[x_{k-1}, X_{k-1}|Z^{k-1}]$, i.e., $\{\hat{x}_{k-1|k-1}, P_{k-1|k-1}, \hat{v}_{k-1|k-1}, \hat{X}_{k-1|k-1}\}$, the Bayesian approach in [2] first predicts the parameters according to dynamic models (3) and (6), and obtains the predicted parameters $\{\hat{x}_{k|k-1}, P_{k|k-1}, \hat{v}_{k|k-1}, \hat{X}_{k|k-1}\}$. Then, based on the measurement Z_k and the likelihood function (10), the updated parameters $\{\hat{x}_{k|k}, P_{k|k}, \hat{v}_{k|k}, \hat{X}_{k|k}\}$ can be calculated using both the measurement values Z_k and the extension-dependent measurement number n_k. For details of the above calculations refer to Table II in [2].

B. The PMHT logic

As described earlier, PMHT adopts a soft decision logic for data association. In particular, the assignments at time k are modeled as discrete random variables $U_k = [u_k^r]_{r=1}^{n_k}$ with the measurement number n_k. Furthermore, each u_k^r with $r \in \{1, 2, ..., n_k\}$ is assumed stochastically independent and identically distributed, and the prior probability mass function (PMF) of each u_k^r can be modeled by parameters $\Pi_k = [\pi_k^m]_{m=1}^M$ [16]. For example, $u_k^r = m$ means the r-th measurement z_k^r is assigned to the m-th object at time k, and $p[u_k^r = m] = \pi_k^m$ for each $r \in \{1, 2, ..., n_k\}$. Note that a fixed number of M objects is assumed here. Then, the unknown parameters Π_k are also required to be estimated, and thus the task of multi-object tracking can be modeled by the optimization problem [16]

$$(\hat{\mathbf{X}}, \hat{\Pi}) \triangleq \arg\max \sum_U p[\mathbf{X}, \Pi, U|Z^K] \tag{12}$$

where the bold capital letter $\mathbf{X} \equiv \mathbf{X}^K = [\mathbf{X}_k]_{k=1}^K$ with $\mathbf{X}_k$ denoting all object states at time k, i.e., $\mathbf{X}_k = [x_k^m]_{m=1}^M$ with x_k^m denoting the state of the m-th object; $\Pi \equiv \Pi^K = [\Pi_k]_{k=1}^K$, $U \equiv U^K = [U_k]_{k=1}^K$, and $Z^K = [Z_k]_{k=1}^K$ with $Z_k = \{z_k^r\}_{r=1}^{n_k}$.

PMHT attempts to track multiple objects by solving (12), and then the object trajectories $\hat{\mathbf{X}}$ can be obtained. In particular, the assignment variable U is regarded as a hidden variable, and then a local optimum of the problem (12) can be obtained by EM which is an iterative batch algorithm. Each iteration includes an E-step and a M-step. Denote the iteration index by i, and then $\mathbf{X}^{(i)}$ and $\Pi^{(i)}$ denote the i-th iteration results. During the $(i+1)$-th iteration, in the E-step, the auxiliary function is calculated based on the i-th iteration results as [16]

$$\begin{aligned} &Q(\mathbf{X}, \Pi; \mathbf{X}^{(i)}, \Pi^{(i)}) \\ &\quad = \sum_U \{\ln p[\mathbf{X}, \Pi, U, Z^K] p[U|Z^K, \mathbf{X}^{(i)}, \Pi^{(i)}]\}. \end{aligned} \tag{13}$$

Then, in the M-step, $(\mathbf{X}^{(i+1)}, \Pi^{(i+1)})$ is evaluated to maximize the auxiliary function (13). The E-step and the M-step are conducted iteratively until the auxiliary function converges or the iteration number reaches a limit I. After that, a local optimum $(\mathbf{X}^{(I)}, \Pi^{(I)})$ can be obtained, and meanwhile the estimated object trajectories $\hat{\mathbf{X}} = \mathbf{X}^{(I)}$ can be output.

III. A New PMHT Approach to MEOT with Extension-Dependent Measurement Numbers

A. General Consideration

As described in Section II-B, the tracking results are obtained by maximizing the auxiliary function which theoretically should contain all available information, such as the object dynamics and the measurement process [18]. Thus, the formulation of the auxiliary function is significantly important.

In [18], to track multiple extended objects, an auxiliary function combined with the RMM in [1] is derived by introducing object extensions into the object variables $\mathbf{X}$, such that $\mathbf{X}_k = [(x_k^m, X_k^m)]_{m=1}^M$. The formulation of the auxiliary function is given as [18]

$$\begin{aligned} Q(\mathbf{X}, \Pi; \mathbf{X}^{(i)}, \Pi^{(i)}) &= \sum_{m=1}^M \{\ln\{p[x_0^m]p[X_0^m]\} \\ &+ \sum_{k=1}^K \ln\{p[x_k^m|x_{k-1}^m, X_k^m]p[X_k^m|X_{k-1}^m]\}\} \\ &+ \sum_{k=1}^K \sum_{r=1}^{n_k} \sum_{m=1}^M \{w_k^{rm(i)} \ln p[z_k^r|x_k^m, X_k^m]\} \\ &+ \sum_{k=1}^K \sum_{r=1}^{n_k} \sum_{m=1}^M [w_k^{rm(i)} \ln \pi_k^m] \end{aligned} \tag{14}$$

with

$$w_k^{rm(i)} = \frac{\pi_k^{m(i)} p[z_k^r|\hat{x}_k^{m(i)}, \bar{X}_k^{m(i)}]}{\sum_{m'=1}^M \pi_k^{m'(i)} p[z_k^r|\hat{x}_k^{m'(i)}, \bar{X}_k^{m'(i)}]} \tag{15}$$

where $\hat{x}_k^{m(i)} = \mathrm{E}[x_k^{m(i)}|Z^k]$ and $\bar{X}_k^{m(i)} = \mathrm{E}[X_k^{m(i)}|Z^k]$.

Note that only the measurement values $\{z_k^r\}_{r=1}^{n_k}$ with $k \in \{1, 2, ..., K\}$ are used, whereas the measurement number of each object is not considered explicitly. Since the measurement number contains the object extension information, a new auxiliary function containing extension-dependent measurement numbers is expected to result in better tracking results.

B. A New Auxiliary Function

In this section, a new auxiliary function combined with the RMM in [2] is derived to consider extension-dependent measurement numbers.

According to the general formulation of the auxiliary function (13), the first term on the right-hand side is called the complete data log-likelihood and can be expanded as

$$\ln p[\mathbf{X}, \Pi, U, Z^K] = \ln\{p[\mathbf{X}]p[U|\mathbf{X}, \Pi]p[Z^K|\mathbf{X}, U]\} \tag{16}$$

with

$$\begin{aligned} p[\mathbf{X}] = & \prod_{m=1}^{M}\prod_{k=1}^{K}\{p[x_k^m|x_{k-1}^m, X_k^m]p[X_k^m|X_{k-1}^m]\} \\ & \times \prod_{m=1}^{M}\{p[x_0^m]p[X_0^m]\}, \end{aligned} \tag{17}$$

$$\begin{aligned} p[U|\mathbf{X}, \Pi] &= \prod_{k=1}^{K} p[U_k|\mathbf{X}_k, \Pi_k] = \prod_{k=1}^{K}\prod_{r=1}^{n_k} p[u_k^r|\Pi_k] \\ &= \prod_{k=1}^{K}\prod_{r=1}^{n_k} \pi_k^{u_k^r}, \end{aligned} \tag{18}$$

$$\begin{aligned} p[Z^K|\mathbf{X}, U] &= \prod_{k=1}^{K} p[Z_k, n_k|\mathbf{X}_k, U_k] \\ &= \prod_{k=1}^{K}\Bigg\{\{\prod_{r=1}^{n_k} p[z_k^r|x_k^{u_k^r}, X_k^{u_k^r}]\} \\ &\quad \times \{\prod_{m=1}^{M} p(\textstyle\sum_{r=1}^{n_k} \mathbb{1}(u_k^r = m); \mu_k, X_k^m)\}\Bigg\} \end{aligned} \tag{19}$$

where $P[\Pi|\mathbf{X}]$ is omitted in (16) since Π is a parameter rather a random variable; (17) holds because the kinematic state and the extension of each object are assumed Markovian and each object is assumed statistically independent; (18) follows from the independent assignments u_k^r with $k \in \{1, 2, ..., K\}$ and $r \in \{1, 2, ..., n_k\}$; $\sum_{r=1}^{n_k} \mathbb{1}(u_k^r = m)$ with the indicator function $\mathbb{1}(\cdot)$ in (19) corresponds to the measurement number assigned to the m-th object. As the Gamma-alike distribution (9) is adopted in (19), the dependency between measurement numbers and extensions is considered explicitly.

For the second term in (13), named posterior assignment weights, it can be evaluated as [18]

$$\begin{aligned} & p[U|Z^K, \mathbf{X}^{(i)}, \Pi^{(i)}] \\ &= \prod_{k=1}^{K}\prod_{r=1}^{n_k} \frac{\pi_k^{u_k^r(i)} p[z_k^r|x_k^{u_k^r(i)}, X_k^{u_k^r(i)}]}{\sum_{u_k^{r\prime}=1}^{M} \pi_k^{u_k^{r\prime}(i)} p[z_k^r|x_k^{u_k^{r\prime}(i)}, X_k^{u_k^{r\prime}(i)}]} \\ &= \prod_{k=1}^{K}\prod_{r=1}^{n_k} w_k^{ru_k^r(i)}. \end{aligned} \tag{20}$$

Substituting (16) and (20) into (13) yields the formulation of the new auxiliary function as (21).

Furthermore, based on the following identities:

$$\begin{aligned} & \sum_{U}\prod_{k=1}^{K}\prod_{r=1}^{n_k} w_k^{ru_k^r} = 1, \\ & \sum_{U}\prod_{k=1}^{K}\prod_{r=1}^{n_k} w_k^{ru_k^r} = \sum_{u_k^{r\prime}}\sum_{U\backslash u_k^{r\prime}}\prod_{k=1}^{K}\prod_{r=1}^{n_k} w_k^{ru_k^r} = \sum_{u_k^{r\prime}} w_k^{ru_k^{r\prime}}, \\ & \sum_{U}\prod_{k=1}^{K}\prod_{r=1}^{n_k} w_k^{ru_k^r} = \sum_{U_{k'}}\sum_{U\backslash U_{k'}}\prod_{k=1}^{K}\prod_{r=1}^{n_k} w_k^{ru_k^r} = \sum_{U_{k'}}\prod_{r=1}^{n_{k'}} w_{k'}^{ru_{k'}^r} \end{aligned} \tag{23}$$

where $\sum_{U\backslash u_k^{r\prime}}$ means the sum over all indices in U except $u_k^{r\prime}$, and $\sum_{U\backslash U_{k'}}$ means the sum over all indices in U except $U_{k'}$, (21) can be further simplified as (22).

Remark 1: Compared with the original auxiliary function (14), the new auxiliary function (22) takes the extension-dependent measurement number of each object into account, by introducing the Gamma-alike distribution [2] and the indicator function $\mathbb{1}(\cdot)$ into the likelihood function part. That is, more information is considered in the new auxiliary function.

Note that the new auxiliary function (22) can be divided into two parts: one is the part of assignment parameters Π and the other is the part of the object variables $\mathbf{X}$. The factorization form of the auxiliary function can be given as

$$\begin{aligned} Q(\mathbf{X}, \Pi; \mathbf{X}^{(i)}, \Pi^{(i)}) = & \sum_{k=1}^{K} Q_{k,\pi}(\Pi_k; \mathbf{X}^{(i)}, \Pi^{(i)}) \\ & + \sum_{m=1}^{M} Q_X^m(x_{1:K}^m, X_{1:K}^m; x_{1:K}^{m(i)}, X_{1:K}^{m(i)}, \Pi^{(i)}) \end{aligned} \tag{24}$$

where the term of single-object variables $(x_{1:K}^m, X_{1:K}^m)$ can be given as

$$\begin{aligned} & Q_X^m(x_{1:K}^m, X_{1:K}^m; x_{1:K}^{m(i)}, X_{1:K}^{m(i)}, \Pi^{(i)}) \propto p[x_0^m]p[X_0^m] \\ & \prod_{k=1}^{K}\{p[x_k^m|x_{k-1}^m, X_k^m]p[X_k^m|X_{k-1}^m]\} \\ & \prod_{k=1}^{K}\{N(\bar{z}_k^{m(i)}; \tilde{H}_k x_k^m, X_k^m/(\textstyle\sum_{r=1}^{n_k} w_k^{rm(i)})) \\ & \times \mathcal{IW}(\bar{Z}_k^{m(i)}; (\textstyle\sum_{r=1}^{n_k} w_k^{rm(i)}) - 1, X_k^m) \\ & \times \prod_{U_k}[p(\textstyle\sum_{r=1}^{n_k} \mathbb{1}(u_k^r = m); \mu_k, X_k^m)\prod_{r=1}^{n_k} w_k^{ru_k^r(i)}]\} \end{aligned} \tag{25}$$

with

$$\begin{aligned} \bar{z}_k^{m(i)} &= [\textstyle\sum_{r=1}^{n_k} (w_k^{rm(i)} z_k^r)]/\textstyle\sum_{r=1}^{n_k} w_k^{rm(i)}, \\ \bar{Z}_k^{m(i)} &= \textstyle\sum_{r=1}^{n_k} [w_k^{rm(i)}(z_k^r - \bar{z}_k^{m(i)})(z_k^r - \bar{z}_k^{m(i)})^{\mathrm{T}}], \end{aligned} \tag{26}$$

$$\begin{aligned} Q(\mathbf{X}, \Pi; \mathbf{X}^{(i)}, \Pi^{(i)}) = & \sum_{U}\prod_{k=1}^{K}\prod_{r=1}^{n_k} w_k^{ru_k^r(i)}\sum_{k=1}^{K}\sum_{r=1}^{n_k}\ln \pi_k^{u_k^r} + \sum_{U}\prod_{k=1}^{K}\prod_{r=1}^{n_k} w_k^{ru_k^r(i)}\sum_{m=1}^{M}\left\{\ln\{p[x_0^m]p[X_0^m]\} + \sum_{k=1}^{K}\ln\{p[x_k^m|x_{k-1}^m, X_k^m]p[X_k^m|X_{k-1}^m]\}\right\} \\ & + \sum_{U}\prod_{k=1}^{K}\prod_{r=1}^{n_k} w_k^{ru_k^r(i)}\sum_{k=1}^{K}\sum_{r=1}^{n_k}\ln p[z_k^r|x_k^{u_k^r}, X_k^{u_k^r}] + \sum_{U}\prod_{k=1}^{K}\prod_{r=1}^{n_k} w_k^{ru_k^r(i)}\sum_{m=1}^{M}\sum_{k=1}^{K} p(\textstyle\sum_{r=1}^{n_k}\mathbb{1}(u_k^r = m); \mu_k, X_k^m). \end{aligned} \tag{21}$$

$$Q(\mathbf{X},\Pi;\mathbf{X}^{(i)},\Pi^{(i)})=\sum_{k=1}^{K}\sum_{r=1}^{n_k}\sum_{m=1}^{M}[w_k^{rm(i)}\ln\pi_k^{u_k^r}]+\sum_{m=1}^{M}\left\{\ln\{p[x_0^m]p[X_0^m]\}+\sum_{k=1}^{K}\ln\{p[x_k^m|x_{k-1}^m,X_k^m]p[X_k^m|X_{k-1}^m]\}\right\}$$
$$+\sum_{k=1}^{K}\sum_{m=1}^{M}\sum_{r=1}^{n_k}\{w_k^{rm(i)}\ln p[z_k^r|x_k^m,X_k^m]\}+\sum_{k=1}^{K}\sum_{m=1}^{M}\sum_{U_k}[p(\textstyle\sum_{r=1}^{n_k}\mathbb{1}(u_k^r=m);\mu_k,X_k^m)\displaystyle\prod_{r=1}^{n_k}w_k^{ru_k^r(i)}]. \tag{22}$$

and the term of single-frame assignment parameters Π_k is

$$Q_{k,\pi}(\Pi_k;\mathbf{X}^{(i)},\Pi^{(i)})=\sum_{r=1}^{n_k}\sum_{m=1}^{M}\ln[\pi_k^m]w_k^{rm(i)}. \tag{27}$$

where $w_k^{rm(i)}$ can be evaluated as (15).

The above factorization form (24) facilitates the derivation of iteration formulae during the M-step. In particular, the object variables $\mathbf{X}^{(i+1)}$ and the assignment parameters $\Pi^{(i+1)}$ can be evaluated separately.

C. Iteration Formulae in EM

In this section, analytical forms of iteration formulae for $(\mathbf{X}^{(i+1)},\Pi^{(i+1)})$ are derived based on the factorization form of the new auxiliary function (24).

First, for object variables $\mathbf{X}^{(i+1)}$, each $(x_{1:K}^{m(i+1)},X_{1:K}^{m(i+1)})$ with $m\in\{1,2,...,M\}$ can be evaluated separately due to the independence assumption, and it aims to maximize (25). However, an analytical form of the iteration formula for $(x_{1:K}^{m(i+1)},X_{1:K}^{m(i+1)})$ cannot be obtained directly according to (25). The reason is that the predicted distribution of the kinematic state and the extension $p[x_k^m,X_k^m|Z^{k-1}]$ is not a conjugate prior with respect to the likelihood function in (25). To construct a conjugacy such that an analytical formula can be obtained, the following approximation is proposed:

$$\begin{aligned}&\prod_{U_k}[p(\textstyle\sum_{r=1}^{n_k}\mathbb{1}(u_k^r=m);\mu_k,X_k^m)\displaystyle\prod_{r=1}^{n_k}w_k^{ru_k^r(i)}]\\&\approx p(\textstyle\sum_{r=1}^{n_k}w_k^{rm(i)};\mu_k,X_k^m).\end{aligned} \tag{28}$$

Based on the approximation (28), the auxiliary function term of single-object variables $(x_{1:K}^m,X_{1:K}^m)$ becomes

$$\begin{aligned}&Q_X^m(x_{1:K}^m,X_{1:K}^m;x_{1:K}^{m(i)},X_{1:K}^{m(i)},\Pi^{(i)})\propto p[x_0^m]p[X_0^m]\\&\prod_{k=1}^{K}\{p[x_k^m|x_{k-1}^m,X_k^m]p[X_k^m|X_{k-1}^m]\}\\&\prod_{k=1}^{K}\{N(\bar{z}_k^{m(i)};\tilde{H}_kx_k^m,X_k^m/(\textstyle\sum_{r=1}^{n_k}w_k^{rm(i)}))\\&\times\mathcal{IW}(\bar{Z}_k^{m(i)};(\textstyle\sum_{r=1}^{n_k}w_k^{rm(i)})-1,X_k^m)\\&\times p(\textstyle\sum_{r=1}^{n_k}w_k^{rm(i)};\mu_k,X_k^m)\}\end{aligned} \tag{29}$$

where the likelihood function at each frame has the same form as the likelihood function of the RMM in [2], i.e., (10). Therefore, the iteration formulae for $(x_{1:K}^{m(i+1)},X_{1:K}^{m(i+1)})$ can be obtained easily according to the Bayesian approach proposed in [2] with $\bar{z}_k$, $\bar{Z}_k$, and n_k replaced with $\bar{z}_k^{m(i)}$, $\bar{Z}_k^{m(i)}$, and $\sum_{r=1}^{n_k}w_k^{rm(i)}$, respectively.

Remark 2: When there is only one object, i.e., $M=1$, $w_k^{rm(i)}=1$ always holds for each $r\in\{1,2,...,n_k\}$, and thus $\bar{z}_k^{m(i)}$, $\bar{Z}_k^{m(i)}$, and $\sum_{r=1}^{n_k}w_k^{rm(i)}$ are exactly equal to $\bar{z}_k$, $\bar{Z}_k$, and n_k, respectively. In this case, the proposed new PMHT approach reduces to the Bayesian approach in [2].

In addition, since EM is a batch algorithm, subsequent measurements can be used to smooth previous object estimates. Thus, the Bayesian smoothing approach for extended objects can be adopted to improve the estimation as (the iteration index $i+1$ is ignored for clarity) [20]

$$\begin{aligned}&\hat{x}_{k|K}^m=\hat{x}_{k+1|K}^m+(Y\otimes I_d)(\hat{x}_{k+1|K}^m-\hat{x}_{k+1|k}^m),\\&P_{k|K}^m=P_{k|k}^m-Y(P_{k+1|k}^m-P_{k+1|K}^m)Y^{\mathrm{T}},\\&\hat{v}_{k|K}^m=\hat{v}_{k|k}^m+(\hat{v}_{k+1|K}^m-\hat{v}_{k+1|k}^m-2(d+1)^2/\delta_k)/\eta,\\&\hat{X}_{k|K}^m=\hat{X}_{k|k}^m+A_k^{-1}(\hat{X}_{k+1|K}^m-\hat{X}_{k+1|k}^m)A_k^{-\mathrm{T}}/(\delta_k\eta),\\&Y=P_{k|k}^mF_k^{\mathrm{T}}(P_{k+1|k}^m)^{-1},\\&\eta=1+(\hat{v}_{k+1|K}^m-\hat{v}_{k+1|k}^m-3(d+1))/\delta_k.\end{aligned} \tag{30}$$

where $\{\hat{x}_{k|K}^m,P_{k|K}^m,\hat{v}_{k|K}^m,\hat{X}_{k|K}^m\}$ denote all estimates of the m-th object at time k conditioned on measurements until time K, i.e., the smoothed estimates at time k; Likewise, $\{\hat{x}_{k+1|K}^m,P_{k+1|K}^m,\hat{v}_{k+1|K}^m,\hat{X}_{k+1|K}^m\}$ represent the smoothed estimates at time $k+1$. Note that the smoothed estimates at time k are based on the smoothed estimates at time $k+1$, so the smoothing procedure is conducted in reverse chronological order, i.e., the estimates at time K are smoothed first.

For assignment parameters $\Pi^{(i+1)}$, each $\Pi_k^{(i+1)}$ with $k\in\{1,2,...,K\}$ also can be evaluated separately. The value of $\Pi_k^{(i+1)}$ should maximize (27) with the constraint $\sum_{m=1}^{M}\pi_k^m=1$. It can be solved easily by introducing a Lagrangian multiplier and has the following form [18]:

$$\pi_k^{m(i+1)}=(\textstyle\sum_{r=1}^{n_k}w_k^{rm(i)})/n_k,\quad m\in\{1,2,...,M\}. \tag{31}$$

In conclusion, a complete iteration procedure for $(\mathbf{X}^{(i+1)},\Pi^{(i+1)})$ is presented in Algorithm 1, based on Bayesian approaches in [2] and [20].

D. The PMHT Approach with Extension-Dependent Measurement Numbers

In this section, the overall procedure of the proposed PMHT approach to MEOT with extension-dependent measurement numbers is elaborated.

1) Initialization: As described earlier, the EM in PMHT is a batch algorithm. Thus, after receiving a new measurement set Z_{K+1} at time $K+1$, a new measurement batch $Z^{K+1}=[Z_k]_{k=1}^{K+1}$ is constructed to estimate $(\mathbf{X}^{K+1},\Pi^{K+1})$. To reduce the computation, a sliding window of a fixed-length L_w can be employed. In particular, only the variables within the last

Algorithm 1: The $(i+1)$-th Iteration Procedure in EM

Input: $\{\pi_k^{m(i)}, \hat{x}_{k|K}^{m(i)}, P_{k|K}^{m(i)}, \hat{v}_{k|K}^{m(i)}, \hat{X}_{k|K}^{m(i)}\}$ with $m \in \{1, ..., M\}$ and $k \in \{1, ..., T\}$, $\{\hat{x}_0^m, P_0^m, \hat{v}_0^m, \hat{X}_0^m\}$ with $m \in \{1, ..., M\}$, measurements Z^K, detection probability P_d, range resolution ρ_r, azimuth resolution ρ_θ, and elevation resolution ρ_α

Output: $\{\pi_k^{m(i+1)}, \hat{x}_{k|K}^{m(i+1)}, P_{k|K}^{m(i+1)}, \hat{v}_{k|K}^{m(i+1)}, \hat{X}_{k|K}^{m(i+1)}\}$ with $m \in \{1, ..., M\}$ and $k \in \{1, ..., T\}$

forall $m \in \{1, 2, ..., M\}$ **do**

  forall $k \in \{1, 2, ..., K\}$ **do**

    forall $r \in \{1, 2, ..., n_k\}$ **do**

      $w_k^{rm(i)} \leftarrow$ (15)

    Update each assignment parameter:

    $\pi_k^{m(i+1)} \leftarrow$ (31)

    Predict the kinematic state and the extension of each object:

$$\hat{x}_{k|k-1}^m = (F_k \otimes I_d)\hat{x}_{k-1|K}^{m(i)}$$
$$P_{k|k-1}^m = F_k P_{k-1|K}^{m(i)} F_k^{\mathrm{T}} + D_k$$
$$\lambda_{k-1} = \hat{v}_{k-1|K}^{m(i)} - 2d - 2$$
$$\hat{v}_{k|k-1}^m = \frac{\delta_k(\lambda_{k-1}+1)(\lambda_{k-1}-1)(\lambda_{k-1}-2)}{\lambda_{k-1}^2(\lambda_{k-1}+\delta_k)} + 2d + 4$$
$$\hat{X}_{k|k-1}^m = \frac{\delta_k}{\lambda_{k-1}}(\hat{v}_{k|k-1}^m - 2d - 2) A_k \hat{X}_{k-1|K}^{m(i)} A_k^{\mathrm{T}}$$

    Update the kinematic state and the extension of each object:

$$(\bar{z}_k^{m(i)}, \bar{Z}_k^{m(i)}) \leftarrow (26)$$
$$S_k = H_k P_{k|k-1}^m H_k^{\mathrm{T}} + |B_k|^{2/d} / \textstyle\sum_{r=1}^{n_k} w_k^{rm(i)}$$
$$K_k = P_{k|k-1}^m H_k^{\mathrm{T}} S_k^{-1}$$
$$G_k \triangleq \bar{z}_k^{m(i)} - (H_k \otimes I_d)\hat{x}_{k|k-1}^m$$
$$\hat{x}_{k|k}^{m(i+1)} = \hat{x}_{k|k-1}^m + (K_k \otimes I_d) G_k$$
$$P_{k|k}^{m(i+1)} = P_{k|k-1}^m - K_k S_k K_k^{\mathrm{T}}$$
$$\hat{v}_{k|k}^{m,0} = \hat{v}_{k|k-1}^m + \textstyle\sum_{r=1}^{n_k} w_k^{rm(i)}$$
$$\hat{v}_{k|k}^{m(i+1)} = \hat{v}_{k|k}^{m,0} + \mu_k$$
$$N_k = S_k^{-1} G_k G_k^{\mathrm{T}}$$
$$\hat{X}_{k|k}^{m,0} = \hat{X}_{k|k-1}^m + N_k + B_k^{-1} \bar{Z}_k^{m(i)} B_k^{-\mathrm{T}}$$
$$\hat{v}_k^1 = \hat{v}_{k|k-1}^m - d - 1$$
$$b_k = 2^{-d/2} d(\hat{v}_k^1 - 1)\Gamma_d[(\hat{v}_k^1 - 1)/2]\Gamma_d[\hat{v}_k^1/2]$$
$$\hat{r}_{k|k-1} = [(\hat{x}_{k|k-1}^m)^{\mathrm{T}} \tilde{H}_k^{\mathrm{T}} \tilde{H}_k \hat{x}_{k|k-1}^m]^{1/2}$$
$$\hat{g}_k = \begin{cases} P_d\pi/(\hat{r}_{k|k-1}\rho_r\rho_\theta), & d = 2 \\ 4P_d\pi/(3\hat{r}_{k|k-1}^2\rho_r\rho_\theta\rho_\alpha), & d = 3 \end{cases}$$
$$\hat{X}_{k|k}^{m(i+1)} = \hat{X}_{k|k}^{m,0} + \frac{2\mu_k(\sum_{r=1}^{n_k} w_k^{rm(i)})\hat{X}_{k|k-1}^m}{\hat{g}_k b_k \left|\hat{X}_{k|k-1}^m\right|^{1/2}}$$

  Smooth the kinematic state and the extension of each object:

$$(\hat{x}_{k|K}^{m(i+1)}, P_{k|K}^{m(i+1)}, \hat{v}_{k|K}^{m(i+1)}, \hat{X}_{k|K}^{m(i+1)}) \leftarrow (30)$$

L_w frames are estimated based on these measurements also within the last L_w frames. For clarity, $K+1 \leq L_w$ is taken here as an example. Then, $(\mathbf{X}^{K+1(0)}, \Pi^{K+1(0)})$ need to be initialized, where the part of $(\mathbf{X}^{K(0)}, \Pi^{K(0)})$ can be obtained directly from the estimation at time K. For $\mathbf{X}_{K+1}^{(0)}$, it can be obtained by prediction using the dynamic models (3) and (6). For $\Pi_{K+1}^{(0)}$, it can be set up as same as $\hat{\Pi}_{K-1}$ for simplicity.

2) Iteration: After initialization, all parameters and variables are updated iteratively according to Algorithm 1. In particular, in each iteration, the assignment parameters Π can be updated easily according to (31), and the object variables $\mathbf{X}$ are first predicted and updated according to the Bayesian approach in [2], then they are smoothed according to the Bayesian approach in [20].

3) Output: The iteration is repeated until the auxiliary function converges or the iteration number reaches a limit I. Then, the estimates $(\mathbf{X}^{K+1(I)}, \Pi^{K+1(I)})$ can be output, of which parameters include $\{\pi_k^{m(I)}, \hat{x}_{k|K+1}^{m(I)}, P_{k|K+1}^{m(I)}, \hat{v}_{k|K+1}^{m(I)}, \hat{X}_{k|K+1}^{m(I)}\}$ with $k \in \{1, 2, ..., K+1\}$ and $m \in \{1, 2, ..., M\}$.

IV. Simulation and Analysis

This section considers a simulated scenario where multiple extended objects exist simultaneously to demonstrate the effectiveness of the proposed new PMHT approach to MEOT, and compares the following four approaches:

1) PMHT-E1: The PMHT approach proposed in [18] (referred to as PMHT-E in [18]) with the extension scale factor $\lambda = 1$ for covariance estimates;

2) PMHT-E2: The PMHT approach proposed in [18] with the extension scale factor $\lambda = 1/4$ for covariance estimates;

3) ET-RM-PMHT: The PMHT approach mentioned in the simulation section in [21] (referred to as ET-RM-PMHT in [21], which combines PMHT with the RMM proposed in [3]);

4) New-PMHT: The new PMHT approach proposed in our work.

In the scenario, four extended objects move as a nearly constant velocity (CV) model with initial states $[-2.5\,\mathrm{m}, 60\,\mathrm{m}, 0\,\mathrm{m/s}, -10\,\mathrm{m/s}]^{\mathrm{T}}$, $[-20\,\mathrm{m}, 37.5\,\mathrm{m}, 10\,\mathrm{m/s}, 0\,\mathrm{m/s}]^{\mathrm{T}}$, $[2.5\,\mathrm{m}, 10\,\mathrm{m}, 0\,\mathrm{m/s}, 10\,\mathrm{m/s}]^{\mathrm{T}}$ and $[20\,\mathrm{m}, 32.5\,\mathrm{m}, -10\,\mathrm{m/s}, 0\,\mathrm{m/s}]^{\mathrm{T}}$. All objects have a length of $5\,\mathrm{m}$ and a width of $2\,\mathrm{m}$, and they are surveilled by a radar fixed at the origin. The radar has a range resolution $\rho_r = 1\,\mathrm{m}$, a range accuracy $\sigma_r = 0.1\,\mathrm{m}$, an azimuth resolution $\rho_\theta = 3\pi/180$rad, and an azimuth accuracy $\sigma_\theta = \pi/180$ rad. For the generation of measurements at each frame, first, polar measurements with Gaussian noises are generated on resolution cells occupied by each object with the detection probability $P_d = 0.3$ [2]. Then, each polar measurement is transformed into the Cartesian coordinate to facilitate the RMM. There are 50 frames in total and the sampling interval $dt = 0.1\,\mathrm{s}$. Fig. 1 displays ground truth trajectories and measurements for one run.

For all compared approaches, a nearly CV model is used with the sigma of the acceleration noise $\sigma_a = 0.5\,\mathrm{m/s^2}$, and the temporal decay constant $\delta_k = 100$. The sliding window size and the maximum iteration number in EM are both 10. The number of objects is assumed as known, and thus the initialization of each object can be done easily by employing a clustering algorithm on the measurements of the first frame. For the ET-RM-PMHT and the New-PMHT, $A_k = I_d/\delta_k^{1/2}$, and $B_k = (1/4\bar{X}_{k|k-1} + R_k^c)^{1/2}\bar{X}_{k|k-1}^{-1/2}$ where the measurement noise covariance in the Cartesian coordinate R_k^c can be obtained approximately as [9]

$$\begin{aligned} R_k^c &= J_k \mathrm{diag}(\sigma_r^2, \sigma_\theta^2) J_k^{\mathrm{T}} \\ J_k &= \begin{bmatrix} \cos\theta_k & -r_k \sin\theta_k \\ \sin\theta_k & r_k \cos\theta_k \end{bmatrix} \end{aligned} \tag{32}$$

where r_k and θ_k can be approximated as the relative distance and the relative azimuth between the object center and the radar, respectively. Moreover, the shape parameter $\mu_k = 0.5$ for the New-PMHT.

The comparison results include the root-mean-square averaged errors of position estimation (referred to as RMSE-x), extension estimation (referred to as RMSE-X), and the root-mean-square of averaged Gaussian Wasserstein Distance (referred to as RMS-GWD). These metrics are evaluated over $M_c = 100$ Monte Carlo runs as follows [7], [22]:

$$\begin{aligned} &\mathrm{RMSE{-}x}_k \\ &= \sqrt{\frac{1}{M_c}\sum_{m_c=1}^{M_c}\left[\frac{1}{M}\sum_{m=1}^{M}\left\|\tilde{H}_k x_{m_c,k}^{m,e} - \tilde{H}_k x_{m_c,k}^{m,t}\right\|_2\right]^2}, \end{aligned} \tag{33}$$

$$\begin{aligned} &\mathrm{RMSE{-}X}_k \\ &= \sqrt{\frac{1}{M_c}\sum_{m_c=1}^{M_c}\left\{\frac{1}{M}\sum_{m=1}^{M}\sqrt{\mathrm{tr}[(X_{m_c,k}^{m,e} - X_{m_c,k}^{m,t})^2]}\right\}^2}, \end{aligned} \tag{34}$$

$$\mathrm{RMSE{-}GWD}_k = \sqrt{\frac{1}{M_c}\sum_{m_c=1}^{M_c}\left[\frac{1}{M}\sum_{m=1}^{M}\mathrm{GWD}_{m_c,k}^{m}\right]^2} \tag{35}$$

with

$$\begin{aligned} \mathrm{GWD}_{m_c,k}^{m} = &\left\|\tilde{H}_k x_{m_c,k}^{m,e} - \tilde{H}_k x_{m_c,k}^{m,t}\right\|_2^2 + \mathrm{tr}[X_{m_c,k}^{m,e} \\ &+ X_{m_c,k}^{m,t} - 2((X_{m_c,k}^{m,t})^{1/2} X_{m_c,k}^{m,e} (X_{m_c,k}^{m,t})^{1/2})^{1/2}] \end{aligned} \tag{36}$$

where $x_{m_c,k}^{m,e}$ and $x_{m_c,k}^{m,t}$ denote the estimated kinematic state and the truth kinematic state for the m-th object at time k during the m_c Monte Carlo run, respectively; Likewise, $X_{m_c,k}^{m,e}$ and $X_{m_c,k}^{m,t}$ denote the estimated extension and the truth extension for the m-th object at time k during the m_c Monte Carlo run, respectively.

The estimated trajectories and the comparison results are presented in Fig. 2 $\sim$ Fig. 6. As shown in Fig. 4, all approaches have similar performances on position estimation. This similarity has been found in [2]. For extension estimation, the proposed New-PMHT outperforms other approaches as shown in Fig. 5, because the extension-dependent measurement number is utilized to improve the extension estimation. A detailed result of extension estimation at the 24th frame is presented in Fig. 3, which is consistent with the result of RMSE-X as the extension estimated by the New-PMHT is closer to the ground truth. Moreover, the proposed New-PMHT also has lower RMS-GWDs as displayed in Fig. 6. Since the RMS-GWD is a comprehensive metric for evaluating the tracking performance of elliptical EOs [22], the effectiveness of the New-PMHT can be demonstrated again.

V. Conclusion

For tracking multiple extended objects, a new PMHT approach combined with a random-matrix model using extension-dependent measurement numbers is proposed. In the proposed approach, not only the measurement values but also the extension-dependent measurement numbers are fully utilized. A new auxiliary function is derived based on both pieces of information, and analytical forms of the iteration formulae for the kinematic state and the extension of each object are obtained approximately using EM. The proposed approach has better tracking performance of MEOT, especially in extension estimation, because the extension information contained in the measurement number is used.

Simulation results demonstrated the effectiveness of the proposed PMHT approach to MEOT, compared with other existing PMHT approaches that use only measurement values. Future research will be focused on extending extension-dependent measurement numbers to more multi-object track-

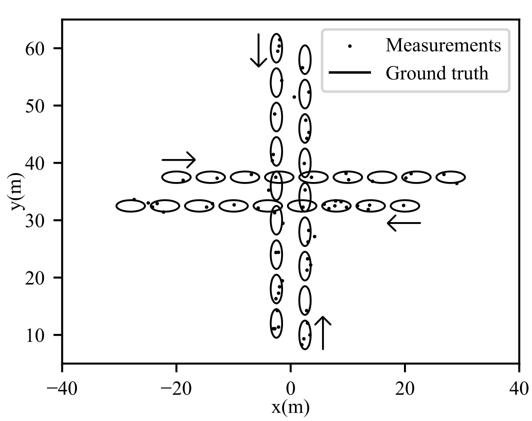

Fig. 1. Ground truth trajectories and measurements for one run.

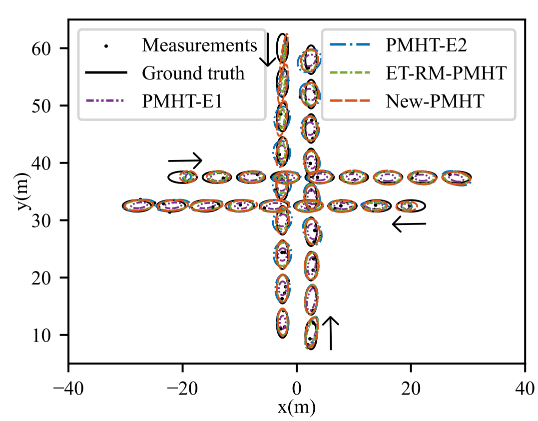

Fig. 2. Object estimated trajectories for one run.

ing approaches, e.g., PMHT with track management and multiple hypothesis tracking (MHT).

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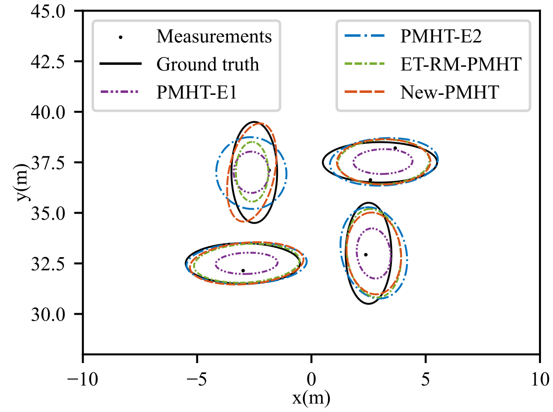

Fig. 3. Object estimated extensions at the 24th frame.

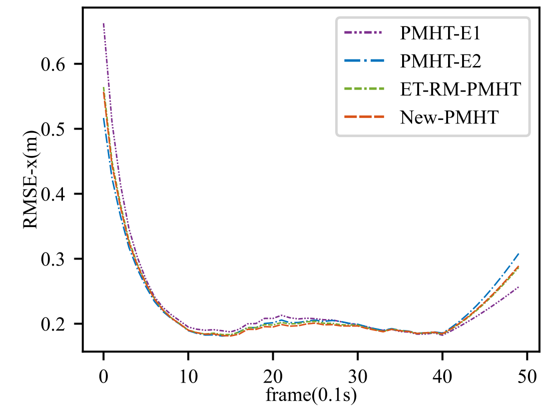

Fig. 4. RMSE-x (m).

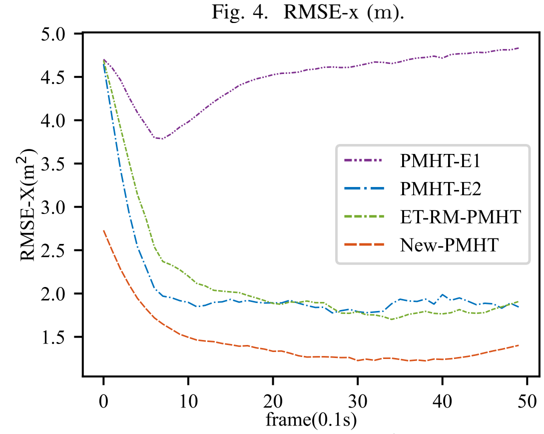

Fig. 5. RMSE-X (m^2).

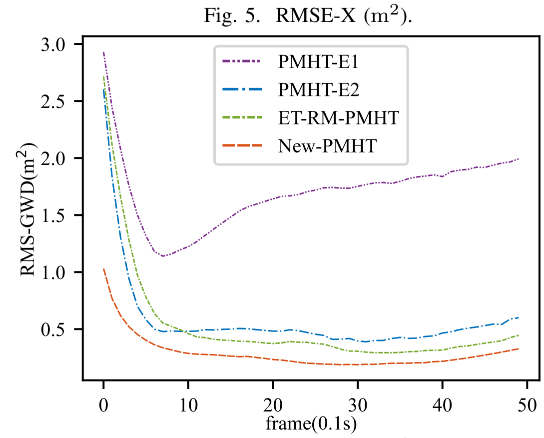

Fig. 6. RMS-GWD (m^2).